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THE MULTIPLE SYSTEM ξ URSAE MAJORIS

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HOVEDKOMMISSIONÆR: ANDR. FRED. HØST & SØN, KGL. HOF-BOGHANDEL
BIANCO LUNOS BOGTRYKKERI
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Introduction.

ξ Ursae majoris is no doubt one of the most interesting stellar systems known at the present time. Its history, since the discovery as a visual double star by Sir William Herschel, is so well known, that it seems unnecessary to give it in detail.

During the first century after this discovery the star did not excite more than usual interest; being a binary in fairly rapid orbital motion, and at all times well measurable with small telescopes, a great number of measures and orbits have been published up till the present.

In 1900 W. H. Wright announced that the radial velocity of the brighter star was variable (Ap. J. **12**, 254).

In 1905 N. E. Nørlund published a discussion of the orbit (A. N. **170**, 117). The great care with which the observations were treated, is evident from the fact that, though eighteen previous orbits had been published, he was the first to discover the perturbation of 1.8 years period, the amplitude being only $0''.05$. Not only did he give the period and epoch of maximum elongation, but we understand from Hertzsprung's publication (A. N. **208**, 111, 1919), that the most striking feature of the small orbit, viz. its inclination of nearly 90°, did not escape him. Evidently Nørlund was unaware of Wright's publication, as he remarks that spectroscopic observations are wanted to confirm this perturbation. He also points out that the best orbit he could derive, based on a least squares solution of not less than 87 normal places, does not give a good fit; in fact there are systematic deviations amounting to nearly 7° in angle near periastron (the distance being $0''.9$) and to $0''.12$ in distance.

In 1908 Wright published (Lick B. **5**, 26), that the radial velocities secured at the Lick Observatory confirmed the period of 1.8 years found by Nørlund. This, as Hertzsprung remarks, is therefore the shortest period derived from micrometer observations of a double star.

In 1914 Hertzsprung started a series of photographic plates with the 50 cm visual refractor of 12.5 m. focal length of the Potsdam Observatory, taking great care to eliminate or reduce the known sources of systematic errors (Publ. Ap. Obs. Potsdam 24_2). He published a provisional discussion of the results 1914—1918 (A. N. **208**, 111), which fully confirmed Nørlund's perturbation.

The first discussion of the Lick radial velocities was published by G. Abetti

(Mem. Spettr. Ital. 8, serie 2[a]) in 1919, in which paper also a list of measures in continuation of Nørlund's list, and a discussion of the areal velocity is given.

The fainter star of the visual system was found to be a spectroscopic binary by Campbell in 1918 (Publ. Ast. Soc. Pacific **30**, 353), so that ξ Ursae Majoris is at least a quadruple system. The material at present available seems to warrant a combined discussion.

The Potsdam plates taken in the years 1914—1923 were kindly put at the writer's disposal by Professor W. Münch, for which courtesy my best thanks are due to him. I am also indebted to Dr. Moore of the Lick Observatory for radial velocities of the brighter star, secured in the years 1897—1924, and to several double star observers for unpublished measures.

The previous investigations indicated at once the most efficient method of treating the observations. Hertzsprung's and Stearns' (A. J. **35**, 157) discussions of photographic results had shown that a circular orbit represented the measures fairly well, whereas Abetti derives an eccentricity of 0.4 from the radial velocities. It was therefore decided to derive the purely elliptical elements e, T and ω from the radial velocities only, and to adopt these results in the discussion of the Potsdam photographic series. In addition the period seemed better determined by the radial velocities, because they cover a much longer interval, and the two earliest as well as the two latest observations are near the narrow maximum of the velocity curve. This leaves the elements a, i and Ω to be determined from the photographic results. If the parallax is known, one of the elements a or i may be found from the radial velocities; or putting it differently, an independent absolute parallax can be derived from the elements a and i as found from the photographic, and $a \sin i$ from the spectroscopic results.

After elimination of the 1.8 year motion from the visual and photographic measures, these may be further used to correct Nørlund's elements of the 60 year system, if necessary.

CHAPTER I.

The Spectroscopic Observations.

In his provisional discussion of the Lick radial velocities 1897—1917, Abetti (l. c.) has neglected the effect of the 60 year motion. To get correct results, it is necessary to take this into account, the amplitude being nearly half that of the 1.8 year motion.

In order to apply this correction we must know:

1st. the parallax

2nd. the mass ratio: $\dfrac{\text{mass of system } B}{\text{total mass}}$

3rd. the elements of the orbit of B round the centre of gravity of A and a

4th. the sign of the inclination of this orbit.

The parallax.

The following determinations of the parallax have been made:

(absolute) trigonometrical parallax

	$+''.169$	$\pm ''.031$ (p. e.)	FLINT; meridian circle (A. J. 27, 50)
	.179	.032	CHASE, ELKIN; heliometer (Yale P. 2, 88)
	.126	.036	ABETTI; meridian circle (Mem. Coll. Rom. 5, 291)
A	.124	.007	STEARNS; Allegheny plates (A. J. 35, 157)
B	.168	.008	

spectroscopic parallax

A	$+''.126$	$\pm ''.023$ (p. e.)	Mount Wilson
B	.120	.022	
	.110		Lockyer
A	.141		Victoria.
B	.093		

An absolute parallax is derived further in this paper from the values $a \sin i$ in Km, a in seconds of arc, and i.

The result is:

$$+''.123 \pm ''.007 \text{ (p. e.)}.$$

The value $0''.130$ has been adopted.

The mass ratio.

The direct determinations of this quantity are not very satisfactory, as they are based on meridian circle observations of a binary with nearly equal magnitudes and a distance scarcely exceeding $3''$ at its maximum. Boss finds from right ascensions 0.59, from declinations 0.43 and adopts 0.50, i. e. equal masses.

ABETTI (l. c.) derives 0.42. Most unfortunately STEARNS, in his discussion of the Allegheny parallax plates, adopts equal masses and uses his material to derive, besides the parallax, the 1.8 year motion in right ascension. It would have been far better to derive the mass ratio instead; it is possible that a better agreement between the parallaxes of the two stars might have resulted. STEARNS' parallaxes differ by $0''.044$ inter se, whereas the probable error of this difference is only $\pm ''.010$, even if we suppose the two parallaxes to be independent (which they are not, being based on the same three comparison stars).

A new reduction of these plates, adopting the relative motions in both the

60 year and 1.8 year orbits from the special determinations thereof, such as the Potsdam photographic and Lick spectroscopic observations, but taking the parallax and mass ratio as the unknowns, might be worth while. Another photographic determination is being made by Professor VAN BIESBROECK at the Yerkes Observatory.

I have tried to strengthen the uncertain direct determinations known at present by an indirect method, based on EDDINGTON's mass-luminosity curve. We suppose that the light of the spectroscopic companions a and b does not materially contribute to the magnitudes of the visual stars, i. e. we assume the magnitudes of A and B to be 4.41 and 4.87, the spectral types $F\,9$ and $G\,1$. The reasons for this supposition are:

the absence of lines of a and b in the spectra of A and B.

the fact that both A and B were found single with the interferometer at Mount Wilson, and that A was never seen double visually.

the fact, so far as A is concerned, that the comparison between Lick radial velocities and Potsdam photographic plates does not show any effect due to the presence of a.

The bolometric magnitudes are 4.37 and 4.77. If the parallax is given, EDDINGTON's curve will give us the masses, but at the same time NØRLUND's elements will furnish the total mass of the system.

We find:

absolute parallax supposed to be $0''.140$.
total mass $1.61\odot$
absolute magnitudes (bolom.) $+5.10$, $+5.50$
masses of A and B $0.90\odot$, $0.82\odot$

and we see that the sum of A and B is already larger than the total mass.

If we take for the absolute parallax $0''.130$, we get:

total mass $2.00\odot$
absolute magnitudes (bolom.) $+4.94$, $+5.34$
masses of A and B $0.94\odot$, $0.85\odot$

leaving for the masses of a and b together $0.21\odot$.

Even this seems rather small; in fact for a alone the value $0.29\odot$ is found later on. However, the reasons for supposing that b is relatively faint, are not nearly as strong as in the case of a, because the short period (9 days according to MOORE's Catalogue), makes it improbable that the pair Bb might be observed even with the interferometer. Therefore in this case we have only the absence of lines in the spectrum to rely upon, which does not exclude the possibility of a difference of magnitudes between B and b as small as 1.5.

EDDINGTON's diagram (Mon. Not. R. A. S. **84**, 311) gives however some indication that the absolutely faint stars have a smaller mass than is indicated by their magnitude. Too much weight can not be attached to these theoretical considerations, but it would seem reasonably safe to infer that the absolute parallax $0''.145$, given as the weighted mean of the trigonometric determinations, is too large, and that the value $0''.130$ adopted in this paper, or perhaps even $0''.123$ derived from the spectroscopic and photographic observations is nearer the truth.

We may get an idea of the mass ratio $\frac{\text{mass } B+b}{\text{total mass}}$ by considering $\frac{0.85}{2.00}$ and $\frac{0.85+0.21}{2.00}$ as limiting values, giving 0.42^5 and 0.53. I adopt 0.46.

Orbital elements of the 60 year motion.

The values $a = 2''.51$, $P = 59.8$ given by Nørlund may be considered as exact compared with the uncertainty of parallax and mass ratio.

The sign of the inclination.

We may now compute the amount of the required correction of the radial velocities, but do not know its sign. The best way to settle this ambiguity is from the radial velocities themselves. It was found that the agreement between the old and the recent observations improved, if we assume Nørlund's node (100°.7) to be the ascending node, or the inclination to be positive; the other alternative showed a marked systematic deviation.

This conclusion is somewhat strengthened by the scarce data published for B. The radial velocity is given to vary between -6 and -18 km./sec. The announcement was made in 1918, and we may take it that the observations were made about 1917. The period is stated to be nearly 9 days, and we may suppose that the excentricity is not large and that the velocity of the centre of gravity of B and b was about -12 km./sec. in 1917. To reduce this to the centre of gravity of the whole system, we have to apply a similar correction as in the case of A, but with opposite sign and a little larger, as we suppose B to have a smaller mass, say, -3 km./sec. in 1917 as compared with $+2.9$ km./sec. for A. We then get -15 km./sec. for the centre of the whole system, agreeing with the value -15.0 derived from the velocities of A. We may therefore consider the sign of the inclination as established; when the radial velocities of B will allow the computation of an orbit, a good mass-ratio will result from a comparison with A.

I have also attempted to strengthen this conclusion in another way. Though the planes of the orbits of Aa and AB do not coincide, the angle is sufficiently small to state that the positive inclination adopted above, would mean that the motions in the 60 year and 1.8 year orbits take place in the same sense. I had some idea that this was the case in most triple systems; in fact, much later I came across Lau's statement (Bull. Astr. **26**, 450): "as the distant companions in triple systems move, *without exception* (italics mine), in the same sense as the close pair ......". This is incorrect, as even at the time Lau made this statement, the system of ξ Scorpionis was known to be an exception. I made up a list of triple systems, but the number which allows a conclusion is still very small. Even if a proper motion which establishes the physical character of the distant companion beyond doubt is known, the orbital motion is nearly always too small with respect to the accuracy of the measures to determine the direction of motion. The result was that 17 out of 21 systems show motion in the same sense and 4 in opposite directions. As this question has some bearing on our theories of the origin of multiple systems, it will be of interest to test this later on, when better data for some distant companions are known, presumably from photographic observation.

For the effect of the 60 year motion on the radial velocity of A we have:

$$+3.80\,[+0.201+\cos(v+309^{\circ}.2)]\text{ km./sec.}$$

The 42 Lick and 4 Bonn observations (the latter reduced to Lick by applying a correction of -1.0) were freed from this effect and a first orbit derived by King's method:

$$\begin{aligned} P &= 670^{d} \\ T &= 2418570\text{ J. D.} \\ e &= 0.50 \\ \omega &= 315^{\circ} \\ K &= 7.8\text{ km./sec.} \\ \gamma &= -14.88\text{ km./sec.} \end{aligned}$$

The representation of the observations by this orbit was already so satisfactory that it was foreseen that a least squares solution would not bring much improvement.

A solution was nevertheless made, introducing besides the corrections Γ, $\varkappa$, π, etc. to the elements a correction ψ to the amplitude of the 60 year effect.

Using, for the sake of homogeneity, only the 42 Lick observations, the normal equations are:

$-0.06 =$	$+42\,\Gamma$	$-16.03\,\varkappa$	$+14.39\,\pi$	$+10.81\,\varepsilon$	$+0.68\,\tau$	$-0.81\,m$	$-20.76\,\psi$
$+1.41$		$+21.38$	$-\ 1.80$	$-\ 1.86$	$+0.63$	$+0.36$	$+\ 6.00$
$+3.17$			$+20.63$	$+\ 3.74$	$+4.98$	$+0.45$	$-\ 6.61$
-2.61				$+\ 6.32$	-0.05	-0.37	$-\ 5.38$
$+1.92$					$+1.82$	$+0.41$	$-\ 0.13$
-0.79						$+1.13$	$+\ 0.34$
-0.82							$+12.91$

The complete solution gives $\psi = -0.38 \pm 0.86$ (m. e.). The mean error of the previous determination of the amplitude can only be estimated. If we take the mean errors of parallax and mass ratio as 10 %, the mean error of our adopted amplitude, 3.80 km./sec., becomes 14 % or ± 0.54. Combining this with the result from the radial velocities we get 3.69 ± 0.46; and a partial solution of the normal equations gives:

$V_1 =$	-11.77	$\pm$ 0.48	(m.e.) + 0.59	$\Delta\psi$
$\gamma =$	-15.01			
$K =$	7.97	$\pm$ 0.37	+ 0.15	$\Delta\psi$
$e =$	0.531	$\pm$ 0.032	+ 0.004	$\Delta\psi$
$\omega =$	320°.0	$\pm$ 5°.6	+ 0°.1	$\Delta\psi$
$T =$	2418582.0 J. D.	$\pm$ 9.1	— 0.7	$\Delta\psi$
or	1909.754	$\pm$ 0.025	— 0.0019	$\Delta\psi$
$P =$	669.18 days	$\pm$ 0.70	+ 0.06	$\Delta\psi$
or	1.8321 year	$\pm$ 0.0019	+ 0.00016	$\Delta\psi$
$n =$	0°.53797 per day	$\pm$ 0°.00056	— 0.00005	$\Delta\psi$

$$a \sin i = 62.2 \pm 2.9 \text{ in } 10^6 \text{ km.}$$

$$\frac{M_a^3 \sin^3 i}{(M_A + M_a)^2} = 0.0214\odot \pm 0.0030\odot.$$

The uncertainty of the 60 year amplitude has no sensible effect on the elements except on V_1 (or γ). The sum of the squares of the residuals has been reduced from 66.4 to 59.6 only. The mean error of a single observation is ± 1.30 km./sec.

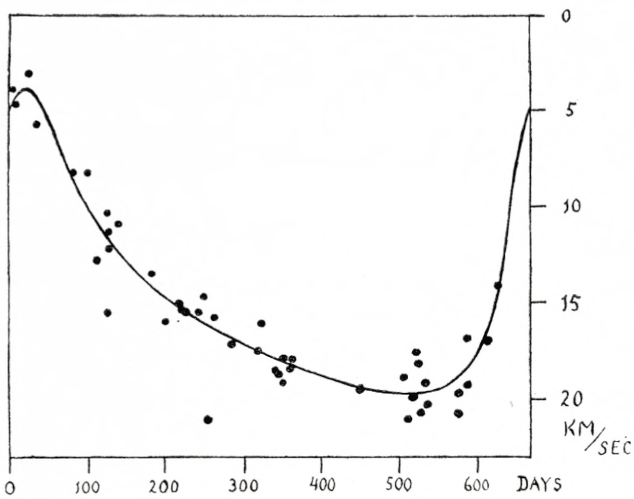


Diagram 1. Radial velocities corrected for 60 year motion.

Table 1 gives the radial velocities and the residuals observed minus computed for the first orbit and the corrected orbit. The 1912 observations were made at Bonn. The column $V_{corr.}$ gives the observed velocity corrected for the effect of the 60 year orbit, however with the amplitude 3.80 km./sec., which was adopted first.

Table 1.

date	J. D. 24 . . .	phase $(t - T)$	$V_{obs.}$	$V_{corr.}$	$O - C$ prelim.	$O - C$ defin.
1897 Febr. 23	13980	82	— 8.4	— 8.2	+ 0.9	+ 0.4
Apr. 8	14024	126	15.7	15.5	— 3.7	— 3.9
1899 Febr. 2	14709	142	11.5	10.9	+ 1.5	+ 1.5
Apr. 5	14751	184	14.1	13.5	+ 0.6	+ 0.7
1900 Febr. 26	15078	511	21.9	21.1	— 1.2	— 1.4
Mar. 9	15089	522	18.4	17.6	+ 2.2	+ 2.1
12	15092	525	19	18.2	+ 1.6	+ 1.5
14	15094	527	21.6	20.8	— 1.0	— 1.1
20	15100	533	20	19.2	+ 0.6	+ 0.4
May 13	15154	587	20.1	19.3	— 1.6	— 0.9
Dec. 10	15364	128	12.2	11.3	+ 0.5	+ 0.4
1901 Apr. 10	15486	250	15.7	14.7	+ 1.3	+ 1.3
May 13	15519	283	18.2	17.2	— 0.4	— 0.4
Dec. 23	15742	506	20.0	18.9	+ 1.0	+ 0.8
1902 Apr. 9	15850	614	18.1	17.0	— 2.1	— 0.6
1903 May 10	16246	341	19.9	18.6	— 0.6	— 0.7
12	16248	343	20.0	18.7	— 0.6	— 0.7
27	16263	358	19.8	18.5	— 0.2	— 0.3
1908 Apr. 8	18041	128	14.2	12.2	— 0.7	— 0.6
June 22	18116	203	18.0	16.0	— 1.4	— 1.3
Nov. 16	18262	349	21.3	19.2	— 1.1	— 1.2
17	18263	350	20.0	17.9	+ 0.2	+ 0.1
29	18275	362	20.1	18.0	+ 0.4	+ 0.2
1909 Febr. 25	18364	451	21.6	19.5	+ 0.1	— 0.1
May 2	18430	517	22.1	20.0	— 0.1	— 0.3
1911 Mar. 21	19118	536	21.6	19.3	+ 0.5	+ 0.3
1912 Mar. 7	19470	219	17.4	15.0	± 0.0	+ 0.2
Apr. 2	19495	244	17.9	15.5	+ 0.3	+ 0.4
13	19506	255	23.5	21.1	— 5.1	— 4.9
21	19514	263	18.2	15.8	+ 0.4	+ 0.6
1916 Febr. 13	20907	318	20.3	17.5	± 0.0	± 0.0
16	20911	321	18.9	16.1	+ 1.2	+ 1.4
1917 Febr. 4	21264	5	6.8	3.9	+ 0.8	+ 0.8
6	21267	8	7.6	4.7	— 0.2	— 0.4
May 10	21360	101	11.2	8.3	+ 1.5	+ 1.6
11	21361	102	11.2	8.3	+ 1.5	+ 1.7
22	21372	113	15.7	12.8	— 2.3	— 2.1
June 5	21386	127	13.2	10.3	+ 1.0	+ 1.2
1921 May 9	22820	223	18.3	15.3	— 0.3	± 0.0
13	22824	227	18.4	15.4	— 0.3	± 0.0
1922 Apr. 25	23171	574	23.8	20.8	— 1.8	— 1.9
27	23173	576	22.7	19.7	— 0.8	— 0.9
1924 Mar. 6	23852	586	19.8	16.9	+ 1.7	+ 1.4
Apr. 17	23894	628	17.2	14.2	± 0.0	+ 0.3
June 22	23960	24	6.0	3.1	+ 1.2	+ 0.7
July 3	23971	35	8.6	5.7	— 0.9	— 1.3

CHAPTER II.

The Photographic Observations.

The plates taken with the 50 cm. visual refractor at Potsdam by Professors Hertzsprung and Münch were measured by the writer in 1923. A description of the methods used in taking and measuring the plates, is given by Hertzsprung in Publik. d. Astrophysikalischen Obs. Nr. 75. In order to get a homogeneous result, the order of measurement of the plates was taken very different from the dates of exposure.[1] A change of personality should therefore have no systematic effect on the resulting elements. Further as a check against such a change, the first three plates measured were re-measured at the end of the series, and the agreement was found as close as possible. About half of the plates were measured with the right eye, the others with the left eye; but each plate, in all four positions, has been measured with the same eye. No systematic difference between the two eyes could be detected by measuring the same plate with both eyes.

The plates taken in the years 1914—1919 had been measured by Hertzsprung; the median difference of his results and mine is $-0''.001 \pm 0''.001$ in $\Delta\delta$, and $+0''.006 \pm 0''.002$ (m. e.) in $\Delta\alpha \cos\delta$, a result which shows how much smaller the effect of personality is in the case of photographic than in the case of visual measures of double stars.

After applying these differences to Hertzsprung's results, the simple mean of the two measurements was taken, as the weights of the same plates never differ greatly.

After rejecting some plates, which had been taken under insufficient conditions, twelve in total, there remained 88 plates, well distributed along the entire period of 1.8 years. The measures were always made with high powers on the microscope, so that the structure of the image is easily seen. Exposures showing deformed structure were not measured. The total number of images measured was 4721 in $\Delta\delta$ film up, 4750 film down, 4740 in $\Delta\alpha \cos\delta$ film up, 4779 film down, giving a total number of 40.000 settings, including the re-measurement. The total internal weight[2], computed from the internal agreement of the images on the same plate, is 2026000 inverted square seconds of arc in $\Delta\delta$ and 1586100 in $\Delta\alpha \cos\delta$. When the results of different plates are compared, it is found that these internal weigths have to be reduced to about a million in each co-ordinate. The incorporation of Hertzsprung's measures of the 1914—1919 plates does not greatly increase the weight. Even for a single setting the mean error of measurement has been found by Hertzsprung and me to be smaller than the mean error due to the image, and as the plate is measured in two positions for each co-ordinate, the increase in weight will be less than 20 %. For the same reason it does not pay to measure the same plate twice, or to make more than a single setting on each image.

[1] As it was uncertain at the time, if I would be able to measure all the plates, the best plates in every year were measured first.

[2] inverted square mean error.

It is worth noting, that when an observer is engaged for some months in measuring photographic plates, making on the average 800 settings a day, not only the speed of measuring increases greatly, but also the accuracy shows a decided improvement. This was proved by comparing the mean errors of the single exposure with Hertzsprung's results for the same plate. Whereas in the beginning my errors slightly exceeded those of Hertzsprung, they became decidedly smaller than his later on. As he has measured the plates in an entirely different order, it is obvious that the accuracy of my settings has increased during the work. The effect would become still more marked if, instead of the total mean errors, the mean errors of measurement only could be compared.

As many measures of the same kind, with the same instrument, had been made by me before 1923, it is likely that this process occurs every time when the observer has been out of practice.

The results of the measurement give the combined motion of the system $B+b$ with respect to the centre of gravity of A and a, and the motion of A with respect to this centre. The separation of the two is most conveniently done by successive approximations. As the elements of the 1.8 year orbit, with the exception of a, i and Ω have already been derived from the radial velocities, it is easy to get a fair approximation to those three:

$$a = 0''.0534 \qquad i = +90^\circ.0 \qquad \Omega = 316^\circ.9\,.$$

By means of these data the 1.8 year effect is sensibly eliminated from the measures, and the rest compared with an ephemeris computed from Nørlund's elements. The residuals could be closely represented by the linear formulae:

$$\begin{aligned} x &= \Delta\delta &&= -0''.025 - 0''.0035\,(t - 1914.0) \\ y &= \Delta\alpha\cos\delta &&= -0''.013 - 0''.0035\,(t - 1914.0)\,. \end{aligned}$$

Adding these corrections to Nørlund's ephemeris the effect of the 60 year motion was now in turn eliminated from the measures and the residuals used for a better approximation to the elements a, i and Ω of the short period:

$$a = 0''.0520 \qquad i = +94^\circ.0 \qquad \Omega = 310^\circ.9$$

after which correction the constant terms in the linear formulae given above were changed to $-0''.0235$ and $-0''.0107$ from the weighted means of the residuals.

These results represent the observations so closely, that it was foreseen, as in the case of the radial velocities, that a least squares solution would not bring a material improvement. A solution was nevertheless made. For this purpose the period of 1.8 years was divided into twelve equal parts, and the plates falling in the same part combined into a normal place. The internal weights of the plates had been computed from the number of exposures and the mean error of the single exposure, and these were accepted as representing the relative weigths.

Hertzsprung (l. c.) reduces his internal weights by constant factors, 0.6 for declinations and 0.8 for right ascensions, but remarks that, if the reduction is to be ascribed to plate error, it would have been better to add a constant number to the square of the mean error, irrespective of the number of exposures on the plate. In this case the internal weights cannot be considered strictly as relative weights.

For every normal place a factor for the reduction of the weight was derived by comparing the simple sum of the internal weights indicated above with the weight derived from the deviations of the separate plates entering into the normal place. As the motion during the interval covered by a normal place was disregarded, these reduction factors may be somewhat on the severe side. They range from 0.22 to 0.93 in declination, and 0.21 to 1.09, with a single exceptional value of 3.85, in right ascension, or in the mean 0.49 and 0.74 respectively. If the motion during the interval covered by a normal place had been allowed for, the results would probably have been very close to HERTZSPRUNG's results 0.6 and 0.8. No reason was found however to change HERTZSPRUNG's device of reducing the weight by a constant factor. Indeed, if the constant plate error was the correct explanation, we should expect the greater weights to show the severer reduction factors. Nothing of the sort was shown however.

The result of the least squares solution was:

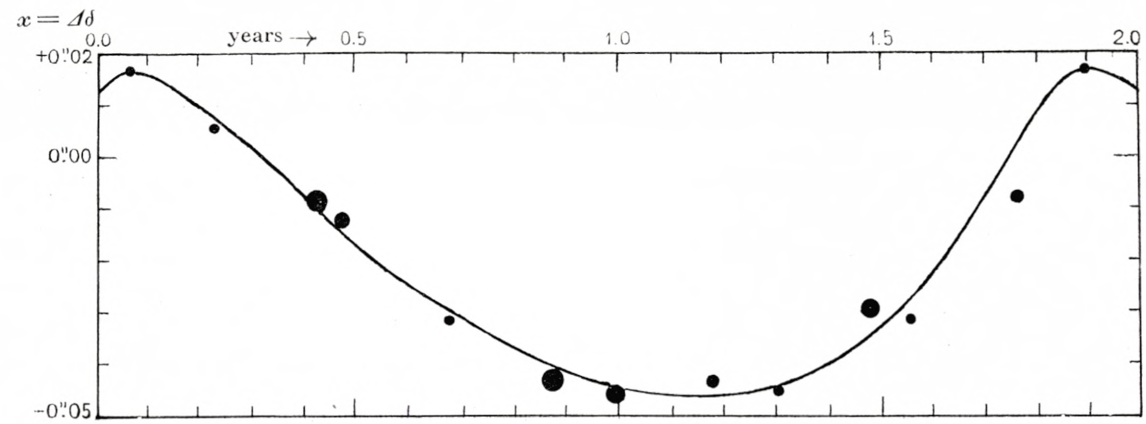
$$
\begin{aligned}
a &= 0''.0514 \pm 0''.0017 \text{ (m. e.)} \\
i &= +\,95^\circ.5 \quad \pm 2^\circ.4 \\
\Omega &= 309^\circ.4 \quad \pm 2^\circ.2
\end{aligned}
$$

The residuals are scarcely improved; the sum of the squares of the weighted residuals is only reduced by 3 per cent.

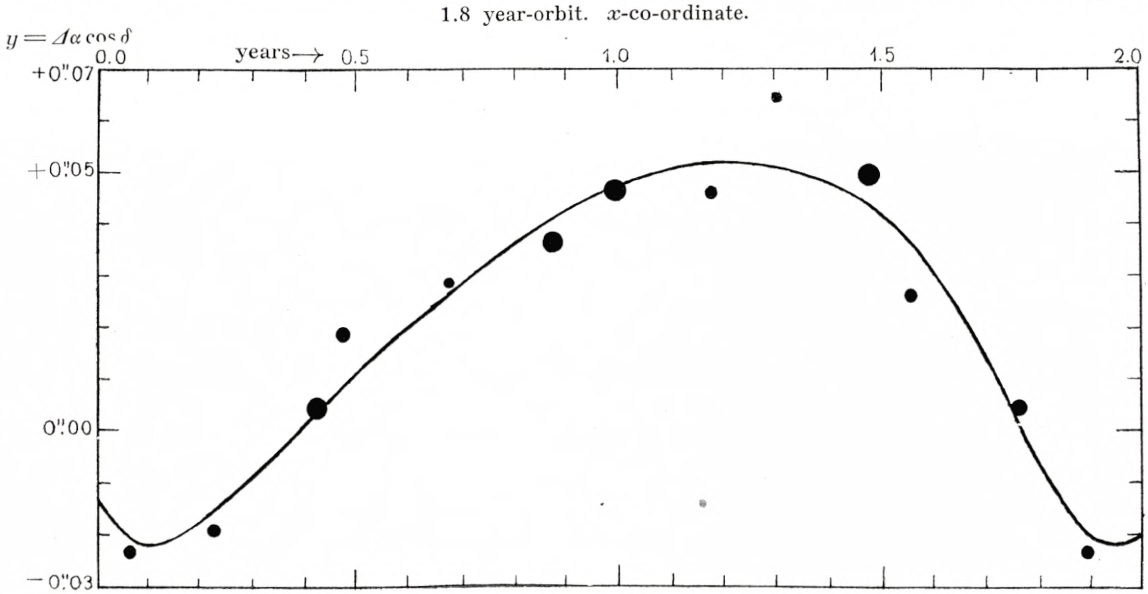
Table IIa. Short period normal places.

a	b	c	d	e	f	g	h	k	l
7	0.063	+ ″.0168	± ″.0047	— ″.0235	± ″.0038	— ″.0004	— ″.0034	+ ″.0003	— ″.0031
5	0.230	+ .0055	47	— .0195	38	— 32	— 51	— 19	— 43
12	0.424	— .0085	26	+ .0041	25	+ 9	— 3	+ 19	+ 5
6	0.477	— .0121	35	+ .0188	34	+ 21	+ 91	+ 29	+ 99
5	0.675	— .0318	47	+ .0287	50	— 21	+ 11	— 19	+ 17
8	0.877	— .0431	26	+ .0355	25	— 21	— 62	— 26	— 59
7	0.993	— .0459	27	+ .0463	25	— 9	— 9	— 17	— 8
5	1.178	— .0433	40	+ .0460	38	+ 38	— 57	+ 25	— 59
6	1.306	— .0452	49	+ .0640	43	— 3	+ 133	— 19	+ 129
15	1.475	— .0294	28	+ .0493	26	+ 71	+ 60	+ 54	+ 54
7	1.556	— .0315	46	+ .0260	42	— 29	— 97	— 46	— 102
5	1.761	— .0080	40	+ .0042	31	— 97	+ 33	— 102	+ 29

column	a	number of plates used in normal place.
—	b	time in years from periastron passage.
—	c,d	x co-ordinate of normal place and its mean error.
—	e,f	y — — — — — — — .
—	g,h	residuals observed minus computed in x and y before, and
—	k,l	after least squares solution.



1.8 year-orbit. x-co-ordinate.



1.8 year-orbit. y-co-ordinate.
(Area of dot proportional to weight of normal place).

The complete set of elements of the orbit of A with respect to the centre of gravity of A and a is:

$\gamma =$	-15.01 km./sec.		
$K =$	7.97 km./sec.	$\pm$ 0.37 km./sec. (m. e.)	
$a =$	$0''.0514$	$\pm 0''.0017$	
$a \sin i =$	62.2	$\pm$ 2.9	in 10^6 km.
$e =$	0.531	$\pm$ 0.032	
$\omega =$	$320^\circ.0$	$\pm 5^\circ.6$	
$i =$	$+95^\circ.5$	$\pm 2^\circ.4$	
$\Omega =$	$309^\circ.4$	$\pm 2^\circ.2$	
$P =$	1.8321	$\pm$ 0.0019	
$n =$	$196^\circ.49$	$\pm 0^\circ.21$	
$T =$	1909.752	$\pm$ 0.025	

INNES' notation (Union Obs. Circ. nr. 68):

$$A = +0''.0274$$
$$B = -0''.0284$$
$$F = +0''.0181$$
$$G = -0''.0279$$
$$C = -0''.0329$$
$$H = +0''.0392$$
$$L = -249 \text{ km./sec.}$$
$$N = +296 \text{ km./sec.}$$

In CAMPBELL's notation we have:

$$i = -84^\circ.5, \text{ angles decreasing}$$
$$\omega = 140^\circ.0$$
$$\Omega = 129^\circ.4$$

From $a \sin i$ and i we find:

$$a = 0.416 \pm 0.019 \text{ astr. units}$$

giving the absolute parallax:

$$\pi = +0''.123 \pm 0''.010 \text{ (m. e.).}$$

Some interesting conclusions are easily derived. By KEPLER's third law we have:

$$\frac{M_a^3}{(M_A + M_a)^2} = \frac{0.0514^3}{1.832^2 \cdot \pi^3}$$

$$M_A + M_a + M_B + M_b = \frac{2.51^3}{59.8^2 \cdot \pi^3}$$

and from the radial velocities:

$$\frac{M_a^3}{(M_A + M_a)^2} = 0.0217 \pm 0.0030 \text{ (m. e.)}$$

which would of course lead to the parallax given above. Hence the total mass of the system becomes

$2.36\odot \pm 0.59\odot$ for the parallax $0''.123$, and
$1.96\odot \pm 0.45\odot$ for the adopted parallax $0''.130$.

With the adopted mass ratio 0.46, and the mass ratio for the short period system as derived from the radial velocities we have:

$$M_A = 0.95\odot,\quad M_a = 0.33\odot,\quad M_B + M_b = 1.08\odot \quad \text{for } \pi = 0''.123$$
$$= 0.77 \qquad\quad = 0.29 \qquad\quad = 0.90 \qquad\qquad\quad = 0''.130$$

and for the relative orbit of a about A:

semi axis major $= 0''.20$, maximum distance $= 0''.28$ for $\pi = ''0.123$
$= 0''.19$ $= 0''.26$ $= ''0.130$.

It has been suspected by NØRLUND that ξ Ursae might be an eclipsing variable. The inclination $95^\circ.5 \pm 2^\circ.4$ makes this improbable, though not impossible, but long ago the sun has been in the orbit plane. The proper motion according to Boss is $0''.733$ in $215^\circ.3$, and as this direction is nearly perpendicular to the node $309^\circ.4$, the proper motion is nearly equal to the change of the inclination. Thus roughly 270 centuries ago (with a mean error of 120 centuries) the sun was in the plane of the orbit, and the brighter component of ξ Ursae was an eclipsing variable.

Table II shows the representation of the Potsdam photographic results 1914—1923, on which the elements were based, and of the Königsberg photographic measures 1923—1926, The latter are from plates taken and measured by Professor E. PRZYBYLLOK with the 13 inch visual refractor, on Schleussner Viridin plates with a yellow filter, and were kindly communicated by him.

The columns give respectively the date, the difference of declination reduced to 1900, same for right ascension, the number of exposures measured in both coordinates, the reduced mean errors of the plate (for Potsdam only; for 1914—1919 these apply to my results, but the x and y are the mean of HERTZSPRUNG's[1] and mine) and the residuals in x and y observed minus computed resulting from the comparison with the elements for the 1.8-year orbit given above, and NØRLUND's elements for the 60-year orbit, adding the linear terms.

Table II. Potsdam Results.

Date	1900		expos.		m. e.		obs.—comp.	
	x	y	x	y	x	y	x	y
1914 .300	$-1''.329$	$+2''.738$	101	115	$\pm''.008$	$\pm''.008$	$+0''.004$	$\pm 0''.000$
.303	.337	.729	191	204	6	6	− 4	− 9
.328	.322	.729	158	136	5	6	+ 8	− 9
.333	.325	.732	72	72	11	11	+ 4	− 6
.369	.324	.730	29	40	22	17	+ 3	− 7
.971	.267	.757	44	34	15	17	+ 18	− 6
.971	.287	.777	47	48	12	10	− 2	+ 14
.974	.258	.755	105	92	8	8	+ 27	− 8
15 .127	.306	.811	47	47	9	8	− 16	+ 5
.185	.299	.834	24	22	10	12	− 5	+ 15
.185	.290	.817	37	38	16	16	+ 4	− 2
.187	.300	.821	45	51	6	4	− 6	+ 1
.187	.294	.823	53	54	10	6	± 0	+ 3
.291	.287	.836	41	40	16	14	+ 9	− 5
.297	.292	.811	32	40	21	11	+ 3	− 31
.300	.308	.834	22	40	13	8	− 12	− 8
.313	.311	.849	47	44	9	10	− 16	+ 5

[1] reduced to my standard by the corrections $+0''.001$ in $\Delta\delta$ and $-0''.006$ in $\Delta\alpha \cos\delta$.

Table II (continued).

Date	1900 x	1900 y	expos. x	expos. y	m. e. x	m. e. y	obs.—comp. x	obs.—comp. y
1915 .313	—1″.298	+2″.840	51	51	±″.013	±″.010	—0″.003	—0″.004
.357	.285	.827	32	30	18	22	+ 5	— 19
16 .083	.155	.812	43	39	11	12	+ 9	+ 4
.083	.158	.794	50	46	12	10	+ 6	— 14
.138	.144	.787	79	75	6	8	+ 11	— 19
.247	.146	.806	94	92	6	6	— 5	+ 3
.247	.145	.802	100	100	5	4	— 4	— 1
.250	.124	.801	48	48	8	5	+ 17	— 2
17 .145	.112	.885	49	38	9	7	— 4	± 0
.238	.087	.874	87	86	9	8	+ 7	— 11
.328	.101	.888	45	42	9	8	— 24	+ 9
.331	.093	.871	54	54	9	8	— 17	— 8
.339	.073	.864	82	84	13	8	+ 2	— 14
.964	0.944	.825	52	50	7	6	+ 15	+ 1
18 .074	.932	.826	53	53	7	7	+ 11	+ 8
.074	.926	.815	48	48	7	7	+ 17	— 3
.210	.916	.823	39	39	12	12	+ 10	+ 11
.210	.893	.805	46	42	11	9	+ 33	— 7
.265	.900	.801	78	59	7	9	+ 20	— 11
.276	.913	.818	47	44	8	7	+ 6	+ 6
.276	.911	.822	45	45	8	7	+ 8	+ 10
19 .251	.855	.853	49	49	8	10	— 7	+ 3
.264	.846	.829	51	44	9	8	± 0	— 19
.335	.838	.828	48	45	10	8	— 8	— 11
.349	.827	.833	55	52	8	8	± 0	— 4
.349	.824	.839	50	50	11	11	+ 3	+ 2
.352	.826	.844	54	54	7	6	+ 1	+ 7
.352	.835	.824	49	51	7	7	— 8	— 13
20 .146	.690	.784	38	38	12	13	+ 7	+ 23
.146	.678	.775	64	62	9	8	+ 19	+ 14
.274	.684	.781	69	68	11	11	± 0	+ 24
.274	.667	.770	23	30	17	16	+ 17	+ 13
.280	.680	.780	32	40	19	9	+ 4	+ 23
.280	.663	.778	60	60	11	13	+ 21	+ 21
.313	.686	.725	14	38	23	16	— 4	— 32
.318	.686	.804	50	49	12	12	— 5	+ 47
.318	.658	.793	27	39	18	12	+ 23	+ 36
.356	.657	.799	19	26	19	16	+ 21	+ 42
.359	.688	.805	33	36	16	14	— 9	+ 48
.373	.669	.791	30	28	14	16	+ 9	+ 33
.389	.711	.794	12	15	35	22	— 34	+ 36
21 .043	.624	.781	16	23	27	18	+ 6	+ 8
.125	.647	.781	75	71	17	17	— 36	+ 21

Table II (continued).

Date	1900 x	1900 y	expos. x	expos. y	m. e. x	m. e. y	obs.—comp. x	obs.—comp. y
1921 .125	—0″.627	+2″.741	24	38	±″.022	±″.017	—0″.016	—0″.019
.196	.643	.716	30	28	18	18	— 47	— 31
.199	.595	.748	68	68	8	7	± 0	+ 1
.199	.593	.742	80	74	7	7	+ 2	— 5
.202	.602	.749	51	45	11	12	— 8	+ 2
.202	.602	.748	83	71	8	8	— 8	+ 1
.224	.579	.743	43	48	9	8	+ 11	± 0
.224	.588	.753	57	63	9	8	+ 2	+ 10
.232	.595	.738	59	52	7	8	— 7	— 4
.232	.584	.753	73	70	9	8	+ 4	+ 11
22 .202	.411	.627	76	77	10	7	+ 29	— 5
.202	.423	.645	77	76	8	8	+ 17	+ 13
.207	.402	.623	65	58	7	8	+ 38	— 9
.207	.456	.643	34	42	17	16	— 16	+ 11
.249	.401	.645	57	50	6	7	+ 37	+ 12
.249	.434	.651	65	60	9	8	+ 4	+ 18
.265	.403	.631	64	58	8	7	+ 33	± 0
.265	.419	.665	30	32	15	16	+ 17	+ 34
.273	.437	.605	45	34	11	15	— 1	— 26
.279	.421	.630	51	55	12	10	+ 16	— 2
.330	.422	.628	26	30	19	14	+ 12	— 4
.336	.452	.668	36	55	17	12	— 18	+ 36
23 .221	.309	.576	49	56	11	10	+ 3	+ 26
.243	.295	.535	65	63	7	8	+ 12	— 11
.243	.327	.530	40	51	11	11	— 20	— 16
.298	.297	.535	42	42	12	13	— 1	± 0
.298	.322	.530	34	30	15	23	— 26	— 5
.341	.281	.515	74	78	10	8	+ 7	— 12

Table II (continued). Königsberg results.

Date	1900 x	1900 y	expos. x	expos. y	obs.—comp. x	obs.—comp. y
1923 .208	—0″.338	+2″.544	11	14	—0″.024	—0″.008
.211	.319	.554	16	17	— 6	+ 2
.301	.322	.560	14	14	— 26	+ 25
.364	.332	.488	16	12	— 48	— 35
.372	.332	.557	14	16	— 50	+ 36
.383	.303	.519	12	10	— 23	± 0

Table II (concluded).

Date	1900		expos.		obs.—comp.	
	x	y	x	y	x	y
1924 .186	—0″.181	+2″.350	18	20	+0″.005	—0″.075
.200	.200	.425	48	52	— 14	± 0
.244	.206	.366	31	35	— 20	— 60
.255	.175	.535	41	35	+ 11	+ 108
.257	.162	.406	34	34	+ 24	— 21
.345	.206	.447	25	28	— 18	+ 15
25 .142	+ .033	.334	20	20	+ 74	+ 54
.222	.038	.270	20	20	+ 64	+ 9
.244	.028	.311	20	20	+ 50	+ 56
.263	.021	.235	10	10	+ 40	— 15
.287	.007	.259	20	20	+ 22	+ 14
.309	.023	.339	20	20	+ 34	+ 98
26 .197	.128	.195	20	20	+ 61	+ 69
.214	.143	.220	20	20	+ 78	+ 92
.227	.148	.233	20	20	+ 82	+ 107
.233	.148	.243	20	20	+ 83	+ 116

The quality of the short-period elements is best judged from the Potsdam results only, as the extrapolation of the linear formulae beyond 1923 in the Königsberg results is not safe and may well be the cause of the systematic deviations for 1925 and 1926.

The residuals in x are systematically positive in 1918 and 1922, those in y equally so in 1920. It is not likely that the short period elements must be blamed for these deviations, as they are contradicted by the results of other years when the phase of the 1.8 year motion was nearly the same, and as they are incorporated in the normal places, from which they could not be removed by the least squares solution. A more probable explanation is failure of the empirical linear formulae to represent the deviations from Nørlund's orbit.

CHAPTER III.

The orbit of long period.

The deviations of Nørlund's normal places from his orbit, though rather too regular, are always small, and the later observations are satisfactorily represented. It is therefore *a priori* doubtful if any substantial improvement on this orbit can be made. On the other hand the great number of visual and photographic observations made or published since seems to warrant the labour of a new discussion, even though the result may afterwards prove to be disappointing.

It is easily seen that, as compared with the spectroscopic and photographic observations, the older micrometer measures cannot give much additional information about the short period elements. They are too far behind in accuracy as well as homogeneity. Therefore the best plan seems to be to remove the short period effect from the measures by means of the elements derived above, and after that to combine the observations into normal places.

Some questions arise at this point. May the short period elements, as derived from material extending from 1897 to 1924, be used for eliminating the effect from the much older visual observations? It will no doubt be interesting to investigate if any measurable perturbation of the short period orbit caused by the visual companion can be detected in future spectroscopic and photographic observations, especially after periastron passage. At present it is however deemed an unnecessary refinement to pay any heed to perturbations, for the following reasons. In the system of ζ Cancri the circumstances are much more favourable for the perturbations to become perceptible than in ξ Ursae. The close pair has a fairly large orbit with the third star relatively near. But even in this case the perturbations are very small. They must be so, *a fortiori*, in ξ Ursae. Moreover even if the short period effect were to be disregarded completely, and the measures of two or four years were combined into normal places, the short period motion would be all but eliminated from these normals, as they would include nearly opposite phases. The long period elements could hardly be affected, and the elimination of the short period motion by means of the constant elements derived earlier, which is in any case a close approximation, is sure to be sufficient for the purpose.

Another point, well known in all investigations on the orbits of well observed double stars, relates to the systematic errors of the micrometer measures. The usual practice of determining personal errors (see AITKEN, The Binary Stars p. 69) is to draw interpolation curves and compare the results of a given observer with the curve in those parts where it is well established. The mean of the differences is then adopted as the personal deviation of this observer from the "mean observer". It is obvious that the application of these corrections has no effect on the well established parts of the curve, as they will bring the measures into harmony with the original curve from which they were derived, even if this had been erroneous. Something may perhaps be said for this device if another part of the curve is based mainly on the results of a single observer having a marked personal deviation. But even in this case we must make the assumption, which especially for the angles is not too safe, that the observer has not changed his personal deviation in the interval. ξ Ursae is so favourably placed for observation, and is always such an easy object, that every part of the orbit has been well observed by a large number of observers. Therefore nothing seems to be gained by the derivation and application of personal deviations. The simple mean of the measures will always closely represent the result of the mean observer. Proceeding now to the systematic error of the mean observer, it is difficult to see a better method of determining this than a comparison of the normal places with the best orbit that can be derived from them, even though this orbit will to some extent be forced into adjustment with these systematically affected normal places. It is true that a comparison of the visual with the photographic results is instructive in this respect, as the systematic errors of the latter, if sensible, are likely to be of an entirely different character, but unfortunately the arc covered by reliable photographic observations is too small at present, and extrapolation is not permissible in this case.

Another question is whether it is sufficient to compare the normal places with an ephemeris derived from constant elements. The positional elements (and also the period and epoch of periastron passage, but this may be disregarded) are affected by the motion of the system relative to the sun, and as the elements of ξ Ursae are determined without ambiguity by the radial velocities, and the proper motion, parallax and radial velocity of the centre of gravity are known, these effects can be calculated. Nørlund's elements and the corresponding positional elements in Innes' notation (see Union Observatory Circular nr. 68) are given below, together with the centennial changes.

P	59.810	
n	$6^\circ.0191$	
T	1935.576	
e	0.4108	
a	$2''.5128$	$100\Delta a + 0''.0005$
i	$+126^\circ.608$	$100\Delta i + 0^\circ.0185$
ω	$129^\circ.213$	$100\Delta\omega - 0^\circ.0105$
Ω	$100^\circ.698$	$100\Delta\Omega - 0^\circ.0136$
A	$+1''.3538$	$100\Delta A - 0''.0005$
B	$-1''.3609$	$100\Delta B - 0''.0001$
F	$-0''.5026$	$100\Delta F - 0''.0008$
G	$-2''.0762$	$100\Delta G - 0''.0006$
C	$+1''.5273$	$100\Delta C + 0''.0004$
H	$-1''.2462$	$100\Delta H - 0''.0004$

The effect of these changes in an extrapolation of 50 years on both sides of Nørlund's mean epoch is at most $\pm 0''.001$ and has been neglected.

It seems unnecessary to take up space by giving the full list of observations used for the present investigation, though it contains some overlooked by Nørlund and many made or published since the time of his or Abetti's list, as nowadays it is easy to collect recent measures by means of the Council Notes in the Monthly Notices of the R. A. S. or the Astronomisches Jahresbericht. All that is wanted are the normal places derived from them.

The observations, taking separate night's results when given, were reduced to

1900, converted into rectangular co-ordinates, corrected for the short period motion and combined into mean results by simple averaging. By means of an ephemeris computed from NØRLUND's elements these means were moved to the nearest normal place. As the epoch of the normals the mean anomalies 0°, $\pm 6^\circ$ etc. up till $\pm 30^\circ$, then with intervals of 12° up till $\pm 90^\circ$ and finally with intervals of 18° till apastron were taken, thus giving time intervals of closely one year near periastron and two or three years when the motion becomes slower.

The weights are based on the number of nights and of observers in the case of the visual measures, adopting as the mean errors of the single night's measure by an average observer $\pm 0''.08$ in angle and $\pm 0''.12$ in distance for a pair of this class, and introducing a factor of 1.2 for more than 10 observers, 1.0 for 6 to 10, and 0.9, 0.8, 0.7, 0.6, and 0.5 for 5, 4, 3, 2 and a single observer respectively. For the photographic measures the reduced weights were adopted for Potsdam, and 0.2 units in each co-ordinate for a single plate Königsberg, the unit of weight being 10000 inverted square seconds of arc, corresponding to a mean error of $\pm 0''.01$.

It may be queried whether this simple averaging is not too crude a method as compared with NØRLUND's way of deriving his normal places, though we may not reasonably ask of a normal place that it should be better than the sum of the observations on which it is based. NØRLUND represented the angles and distances (or their residuals from a preliminary orbit), plotted against the time, by smooth curves which he further corrected by means of the law of areas, and read of his normal places from these curves. It is probable that the accidental errors and partly also the systematic errors are greatly reduced by this process, but there are two drawbacks to this method. The first is that we may get away from the observations by overadjustment. The second is that the normal places taken from these adjusted interpolation curves do not necessarily fall on an ellipse.

If they do, as they are in agreement with the law of areas, the least squares solution will give an orbit practically identical with the interpolation curves. If they do not, which is the more probable case, the solution, with only seven unknowns available, will try to adjust one smooth curve, the orbit, to another smooth curve, the overadjusted normal places. The residuals are likely to be small but very regular, and we cannot expect many changes of sign. This is exactly what NØRLUND's residuals show. On the other hand when the normal places are simply means of observations, we may expect larger but irregular residuals, unless either the motion is not truly Keplerian, or the normal places are vitiated by systematic errors. Investigation of the character of the residuals will show which alternative is the more probable.

Table III shows the normal places finally arrived at. The columns give the date, the mean anomaly, the observed x-co-ordinate or difference of declination, the residual observed minus computed from NØRLUND's orbit and from the elements to be given later, the weight in the unit specialised above, the same data for the other co-ordinate, the number of single night's observations in angle and distance, and the number of observers. For the last five normal places the visual and photographic results are given separately below. The total number of visual measures used (the few scattered photographic observations before 1914 as well as those by STEARNS on Allegheny parallax plates have been taken as visual measures) is 2752 angles and 2501 distances, giving a total weight of 32.60 units in x, and 27.84 in y. The few position angles before 1823 and also some observations by TALMAGE and WALDO which give impossible deviations, have been rejected. The weight of the Potsdam and Königsberg measures is 206.40 in x and 162.41 in y or already now about six times that of all the visual measures, though the weight of the latter is more likely to be over- than underestimated. A future orbit will be based chiefly on photographic observations, except near periastron, where the distance is too small, unless a very long focus (Barlow lens) can be used. At present the arc covered by the photographic observations is however so small that they will mainly determine the node and semi axis. Comparing the visual and photographic results we find no evidence of systematic error in angle in this part of the orbit, where the line joining the stars is nearly horizontal, a tendency of the average visual observer to measure the distance larger than the photographic results, and a confirmation of the supposed superior accuracy of angles over distances.

Before submitting these residuals to a least squares solution their character was studied in two different ways. First they were plotted against the time, as in this way an unknown perturbation is easily detected. No deviations of a periodic character were however revealed. As far as the accuracy of the earlier normal places goes there is a tendency for the deviations to repeat themselves after a revolution. Then the deviations were plotted against a diagram of NØRLUND's orbit. This will show better a dependence of the residuals on the position in the orbit. The large deviations of the earlier normal places are nearly all in distance, the older observers measuring too large. In the angles there may perhaps be a tendency to measure too near the horizontal line when the angle is near 90 or 270°, and too near the vertical when the angle is near 0 or 180°, but the effect is not shown with certainty. In theory systematic errors should be corrected before proceeding to a least squares solution. This is however scarcely possible here. Where the deviations are really serious, near periastron, their character is clearly not that of systematic errors of measurement. Through nearly two quadrants the observed positions, though close to the orbit, are constantly ahead of the computed. Near apastron the weight of the visual measures vanishes against the photographic. The discordant early distances have insignificant weights. A systematic error depending on the position angle is too doubtful, as was expected *a priori*. In cases as 70 Ophiuchi, where the observations are always made near the meridian, such assumptions may be made, as has been done by LAU and LOHSE. But ξ Ursae is frequently observed far from the meridian because of its higher declination, and then the apparent position angle, on which the error really depends, differs too much from the true angle.

Table III.

t	M	x	$O-C$ I	$O-C$ II	W_x	y	$O-C$ I	$O-C$ II	W_y	p	d	obs.
	°	″	″	″		″	″	″				
1824.927	+ 54	− 0.93	− .23	− .20	.08	− 2.17	− .58	− .56	.04	11	7	3
1826.921	66	1.06	− .06	− .03	.04	1.37	− .03	− .03	.04	6	7	1
1828.915	78	1.40	− .14	− .12	.04	1.05	− .01	− .02	.05	9	8	2
1830.908	90	1.719	− .234	− .223	.27	0.750	− .058	− .063	.65	64	40	5
1833.899	108	1.875	− .140	− .134	.16	− 0.189	− .049	− .049	.57	38	23	6
1836.889	126	2.126	− .231	− .225	.13	+ 0.428	+ .008	+ .014	.36	26	20	5
1839.880	144	2.074	− .099	− .092	.41	0.947	− .015	− .005	.68	55	51	10
1842.870	162	2.052	− .074	− .064	.59	1.475	+ .008	+ .020	.74	70	69	9
1845.861	180	1.931	− .021	− .007	.62	1.866	− .054	− .040	.62	71	61	6
1848.852	− 162	1.780	− .005	+ .013	.41	2.235	− .070	− .055	.36	41	39	6
1851.842	144	1.598	− .022	− .001	1.32	2.502	− .106	− .090	.89	101	89	12
1854.833	126	1.357	− .040	− .018	.80	2.761	− .049	− .031	.40	81	50	9
1857.823	108	0.991	+ .010	+ .031	.76	2.811	− .079	− .058	.40	57	54	6
1860.814	90	0.634	+ .001	+ .017	.64	2.888	+ .066	+ .095	.25	45	35	7
1862.807	78	0.366	+ .001	+ .013	.69	2.732	+ .053	+ .093	.30	45	43	8
1864.801	66	− 0.096	− .009	− .003	.63	2.388	− .053	+ .001	.28	45	44	5
1866.795	54	+ 0.201	+ .005	+ .005	.83	2.100	+ .005	+ .078	.30	54	42	10
1868.788	42	0.483	+ .016	+ .011	.41	1.532	− .095	+ .008	.20	30	27	8
1870.782	30	0.750	+ .051	+ .046	.31	0.987	− .040	+ .095	.19	31	30	4
1871.779	24	0.841	+ .054	+ .053	.22	0.579	− .103	+ .048	.30	28	26	9
1872.776	18	1.002	+ .155	+ .160	.34	+ 0.212	− .101	+ .063	.78	55	48	10
1873.772	12	0.887	+ .014	+ .030	.17	− 0.175	− .106	+ .063	.46	27	19	11
1874.769	6	0.879	+ .022	+ .049	.17	0.564	− .117	+ .050	.24	22	20	6
1875.766	0	0.787	− .011	+ .028	.30	0.919	− .117	+ .037	.29	38	29	9
1876.763	+ 6	0.727	+ .031	+ .080	.47	1.294	− .180	− .046	.30	46	35	9
1877.760	12	0.547	− .011	+ .044	.48	1.501	− .132	− .022	.29	30	32	11
1878.756	18	0.295	− .099	− .042	.05	1.645	− .085	± 000	.02	6	6	2
1879.753	24	0.247	+ .034	+ .088	.44	1.822	− .135	− .072	.19	29	27	10
1880.750	30	+ 0.089	+ .064	+ .114	.70	1.836	− .081	− .036	.30	45	43	10
1882.744	42	− 0.354	− .005	+ .034	1.19	1.904	− .158	− .138	.53	66	62	11
1884.737	54	0.734	− .038	− .011	.82	1.701	− .108	− .101	.41	67	54	10
1886.731	66	1.022	− .019	− .001	.51	1.379	− .034	− .031	.40	48	46	8
1888.725	78	1.331	− .065	− .054	.82	1.103	− .066	− .063	.95	76	75	11
1890.718	90	1.484	+ .001	+ .007	.59	0.704	− .013	− .007	.97	66	62	16
1893.709	108	1.757	− .022	− 020	.83	− 0.154	− .014	− .004	1.88	105	98	19
1896.699	126	1.948	− .053	− .049	.84	+ 0.402	− .018	− .002	1.82	102	98	22
1899.690	144	1.986	− .011	− .005	.89	0.980	+ .018	+ .037	1.51	99	96	21
1902.680	162	1.999	− .021	− .010	.91	1.494	+ .027	+ .047	1.15	91	88	18
1905.671	180	1.863	+ .047	+ .063	1.73	1.827	− .093	− .072	1.76	157	148	17
1908.662	− 162	1.778	− .003	+ .018	1.38	2.262	− .043	− .022	1.09	114	105	22
1911.652	144	1.621	− .045	− .020	1.31	2.613	+ .005	+ .026	.89	97	91	16
1914.643	126	1.3427	− .0258	+ .0014	43.84	2.7907	− .0188	+ .0011	38.46			
1917.633	108	− 1.0295	− .0283	− .0014	64.30	+ 2.8662	− .0239	− .0024	49.95			

Table III (concluded).

t	M	x	$O-C$ I	$O-C$ II	W_x	y	$O-C$ I	$O-C$ II	W_y	p	d	obs.
1920.624	-90°	$-0.6809''$	$-.0463''$	$-.0226''$	61.35	$+2.7961''$	$-.0262''$	$+.0006''$	45.92			
1922.617	78	0.4079	$-.0409$	$-.0211$	39.17	2.6440	$-.0346$	$-.0003$	27.95			
1924.611	66	0.116	$-.029$	$-.015$	6.04	2.403	$-.038$	$+.008$	4.12			
1914 vis.			$-.038$	$-.011$	.96		$-.022$	$-.002$	.60	58	58	14
phot.			$-.0255$	$+.0017$	42.88		$-.0187$	$+.0012$	37.86			
1917 vis.			$-.029$	$-.002$	1.14		$\pm .000$	$+.022$	.60	65	65	13
phot.			$-.0283$	$-.0014$	63.16		$-.0242$	$-.0027$	49.35			
1920 vis.			$-.053$	$-.029$	1.24		$+.038$	$+.065$	.60	69	67	18
phot.			$-.0462$	$-.0225$	60.11		$-.0270$	$-.0002$	45.32			
1922 vis.			$-.056$	$-.036$	1.52		$+.097$	$+.131$	.67	83	81	11
phot.			$-.0403$	$-.0205$	37.65		$-.0378$	$-.0035$	27.28			
1924 vis.			$-.036$	$-.022$	3.44		$-.087$	$-.041$	1.52	183	183	14
phot.			$-.019$	$-.005$	2.60		$-.009$	$+.037$	2.60			

An investigation would therefore have to be made of separate night's results for separate observers. It could only be carried out for part of the measures, when the hour angle has been given, and if the observer has made a sufficient number of measures. The doubtful succes to be obtained would not warrant the amount of work involved in computing the parallactic angles and discussing the separate nights.

A least squares solution was therefore made taking the residuals and their weights as they stand. The method described in Union Observatory Circular 68 has been used.

Introducing the unknowns:

$$10\,\Delta A = p \quad 10\,\Delta B = q \quad 10\,\Delta F = r \quad 10\,\Delta G = t \quad 100\,\Delta e = u \quad n\,\Delta T = v \quad 10\,\Delta n = w$$

the normal equations in the x-co-ordinate are:

$$\begin{array}{rrrrrcr}
+2.045\,p & +1.416\,r & +0.500\,u & +0.383\,v & -1.587\,w & = & +0.694 \\
 & +1.377 & +0.422 & +0.359 & -1.471 & & +0.574 \\
 & & +0.149 & +0.110 & -0.478 & & +0.201 \\
 & & & +0.095 & -0.383 & & +0.152 \\
 & & & & +1.708 & & -0.691
\end{array}$$

and in y:

$$\begin{array}{rrrrrcr}
+1.707\,q & +1.053\,t & -0.154\,u & +0.039\,v & +0.088\,w & = & +0.410 \\
 & +1.057 & -0.088 & -0.088 & +0.301 & & +0.313 \\
 & & +0.028 & +0.002 & -0.039 & & -0.060 \\
 & & & +0.038 & -0.074 & & -0.028 \\
 & & & & +0.411 & & +0.108
\end{array}$$

Solution:

$$\begin{aligned}
\Delta A &= +0''.0219 \pm 0''.0074 \text{ (mean errors)}\\
\Delta B &= +0''.0753 \quad 0''.0189\\
\Delta F &= +0''.0883 \quad 0''.0181\\
\Delta G &= -0''.0683 \quad 0''.0233\\
\Delta e &= +\ 0.0020 \quad 0.0043\\
n\Delta T &= -\ 3^\circ.12 \quad 0^\circ.71\\
\Delta n &= -\ 0^\circ.0054 \quad 0^\circ.0098
\end{aligned}$$

The sum of the squares of the weighted residuals has been reduced from 0.351514 to 0.125740 square seconds of arc in the x-, and from 0.237816 to 0.078907 in the y-co-ordinate, or a reduction by 64 and 67 percent respectively.

The mean error of the unit weight is found to be $\pm 0''.049$. That this is much larger than the assumed value $\pm 0''.010$, on which the weights were based, is probably explained both by the systematic errors of the normal places and by overestimation of the accuracy of the measures.

The resulting elements, with Nørlund's for comparison, are:

$P =$	59.863	$\pm$ 0.098	(mean errors)	59.810
$n =$	$6^\circ.0137$	$0^\circ.0098$		$6^\circ.0191$
$T =$	1935.027	0.118		1935.576
$e =$	0.4128	0.0043		0.4108
$A =$	$+1''.3757$	$0''.0074$		$+1''.3538$
$B =$	$-1''.2856$	$0''.0189$		$-1''.3609$
$F =$	$-0''.4143$	$0''.0181$		$-0''.5026$
$G =$	$-2''.1445$	$0''.0233$		$-2''.0762$
$C =$	$+1''.6982$			$+1''.5273$
$H =$	$-1''.2879$			$-1''.2462$
$a =$	$2''.5355$			$2''.5128$
$i =$	$+122^\circ.801$	(angles decreasing)		$+126^\circ.608$
$\omega =$	$127^\circ.176$			$129^\circ.213$
$\Omega =$	$101^\circ.400$			$100^\circ.698$
				(Nørlund)

equinox 1900.

Comparing the residuals of the normal places in table III from the new orbit with those from Nørlund's orbit, it seems that the least squares solution has been worth while. As a consequence of the earlier periastron passage the observations near periastron are better represented, in fact the greater part of the remaining residuals is in the distance and due to overmeasurement when the pair gets close. There is no reason to suspect real deviations from Keplerian motion. In ten years time the uncertainty of the elements, notably that of T, will be greatly reduced, but even now ξ Ursae is certainly one of the best known double star orbits. The photographic observations are well represented, except the declinations for 1920 and 1922. It seems difficult to blame the elements for this, because of the small deviations of 1914 and 1917 and the right ascensions. A possible cause may be the method of measurement in rectangular co-ordinates. When in a close double star

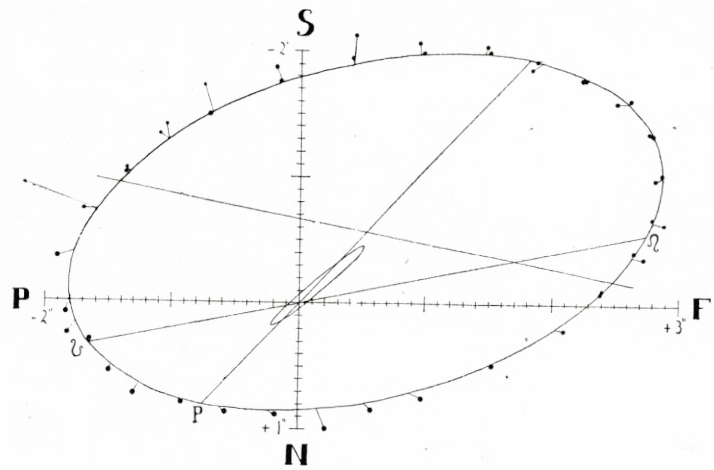


Diagram 3. Apparent orbit of B about $A + a$, with a tenfold magnification of the apparent orbit of A about $A + a$ inside. The normal places for the orbit of B are joined with the computed places; and the line of nodes, projected axis major and perpendicular diameter of the auxiliary circle of this orbit are shown.

the position angle is near 90° it is difficult to make the settings on both stars in declination independent of each other. In any case a similar effect was found at first in the 1925 Königsberg observations, but much more pronounced because of the smaller focal length and the still smaller distance. This effect disappeared when Prof. PRZYBYLLOK had remeasured the plates in polar co-ordinates in a new measuring instrument. The obvious way to settle this question is a remeasurement of the later Potsdam plates in polar co-ordinates.

Because of the combined effect of the two orbits it is most convenient to give an ephemeris in rectangular co-ordinates. Table IV gives the co-ordinates of A with respect to the centre of gravity of A and a, table V those of B. Subtracting the result of table IV from that of table V gives the co-ordinates to be compared with observation.

For the precession we have:

$$p_t = p_{1900} + 0^\circ.0013\ (t - 1900)$$
$$x_t = x_{1900} - 0.000023\, y\, (t - 1900)$$
$$y_t = y_{1900} + 0.000023\, x\, (t - 1900)$$

Table IV.

M	$t—T$	x	y	values of T
$+\ 0^\circ$	0.000	$+0''.0129$	$-0''.0133$	1909.752
15	0.076	169	214	1911.584
30	0.153	137	206	1913.416
45	0.229	77	154	1915.248
60	0.305	+ 6	83	1917.080
75	0.382	— 65	— 7	1918.912
90	0.458	134	+ 70	1920.745
105	0.534	197	145	1922.577
120	0.611	256	214	1924.409
135	0.687	307	279	1926.241
150	0.763	351	338	1928.073
+ 165	0.840	389	391	1929.905
180	0.916	419	435	1931.737
— 165	0.992	441	471	1933.569
150	1.069	455	498	1935.401
135	1.145	459	515	1937.234
120	1.221	454	520	1939.066
105	1.298	437	513	
90	1.374	406	490	
75	1.450	363	451	
60	1.527	300	389	
45	1.603	219	302	
30	1.679	— 113	180	
15	1.756	+ 13	+ 26	
— 0	1.832	129	— 133	

Table V.

M	X	Y	t	x	y	M	t	x	y
$+\ 0^\circ$	+ 0.5872	+ 0.0000	1875.164	$+0''.8078$	$-0''.7549$	-180°	1905.095	$-1''.9436$	$+1''.8163$
3	.5832	.0813	.663	.7688	.9236	177	.594	.9286	.8879
6	.5714	.1610	76.162	.7194	1.0799	174	06.093	.9119	.9574
9	.5522	.2388	.661	.6607	.2221	171	.592	.8932	2.0252
12	.5260	.3137	77.159	.5936	.3490	168	07.090	.8727	.0910
15	.4936	.3848	.658	.5196	.4598	165	.589	.8503	.1546
18	.4556	.4520	78.157	.4395	.5549	162	08.088	.8262	.2153
21	.4129	.5138	.656	.3551	.6327	159	.587	.8002	.2756
24	.3662	.5711	79.155	.2672	.6955	156	09.086	.7723	.3328
27	.3163	.6233	.654	.1769	.7434	153	.585	.7428	.3875
30	.2639	.6706	80.152	+ .0852	.7773	150	10.084	.7114	.4398
33	.2095	.7129	.651	— .0072	.7982	147	.582	.6783	.4896
36	.1537	.7505	81.150	.0994	.8071	144	11.081	.6433	.5368
+ 39	+ 0.0970	+ 0.7835	.649	— 0.1912	— 1.8050	— 141	.580	— 1.6067	+ 2.5813

Table V (concluded).

M	X	Y	t	x	y	M	t	x	y
+ 42°	+ 0.0398	+ 0.8122	1882.148	— 0.″2818	— 1.″7928	— 138°	1912.079	— 1.″5684	+ 2.″6230
45	— .0177	.8367	.647	.3710	.7715	135	.578	.5282	.6618
48	.0751	.8573	83.146	.4585	.7419	132	13.077	.4864	.6977
51	.1322	.8742	.644	.5440	.7048	129	.575	.4429	.7305
54	.1887	.8876	84.143	.6274	.6609	126	14.074	.3978	.7601
57	.2446	.8978	.642	.7084	.6109	123	.573	.3509	.7864
60	.2996	.9049	85.141	.7870	.5555	120	15.072	.3024	.8093
63	.3537	.9092	.640	.8632	.4951	117	.571	.2523	.8287
66	.4067	.9108	86.139	.9368	.4303	114	16.070	.2006	.8444
69	.4586	.9098	.638	1.0079	.3616	111	.569	.1472	.8564
72	.5094	.9065	87.136	.0763	.2892	108	17.067	.0924	.8645
75	.5588	.9010	.635	.1421	.2138	105	.566	.0359	.8686
78	.6070	.8934	88.134	.2053	.1356	102	18.065	0.9780	.8684
81	.6539	.8839	.633	.2658	.0549	99	.564	.9184	.8638
84	.6995	.8726	89.132	.3238	0.9720	96	19.063	.8577	.8550
87	.7436	.8595	.631	.3791	.8872	93	.562	.7955	.8413
90	.7864	.8448	90.130	.4319	.8008	90	20.060	.7318	.8228
93	.8278	.8287	.628	.4821	.7129	87	.559	.6669	.7993
96	.8678	.8111	91.127	.5298	.6238	84	21.058	.6008	.7705
99	.9062	.7922	.626	.5748	.5338	81	.557	.5334	.7363
102	.9434	.7720	92.125	.6177	.4427	78	22.056	.4649	.6964
105	.9791	.7507	.624	.6580	.3511	75	.555	.3955	.6507
108	1.0134	.7282	93.123	.6958	.2589	72	23.054	.3252	.5989
111	.0462	.7048	.621	.7312	.1665	69	.552	.2540	.5408
114	.0776	.6804	94.120	.7644	— .0737	66	24.051	.1822	.4760
117	.1076	.6551	.619	.7951	+ .0191	63	.550	.1099	.4045
120	.1361	.6289	95.118	.8235	.1119	60	25.049	— .0372	.3258
123	.1633	.6020	.617	.8497	.2046	57	.548	+ .0355	.2398
126	.1890	.5743	96.116	.8736	.2970	54	26.047	.1081	.1461
129	.2133	.5459	.615	.8953	.3890	51	.546	.1803	.0446
132	.2362	.5169	97.113	.9148	.4807	48	27.044	.2518	1.9350
135	.2576	.4873	.612	.9320	.5718	45	.543	.3223	.8170
138	.2777	.4572	98.111	.9472	.6622	42	28.042	.3912	.6906
141	.2964	.4265	.610	.9601	.7519	39	.541	.4581	.5556
144	.3136	.3954	99.109	.9710	.8408	36	29.040	.5224	.4118
147	.3295	.3639	.608	.9798	.9288	33	.539	.5836	.2595
150	.3440	.3320	1900.106	.9865	1.0159	30	30.038	.6408	.0988
153	.3571	.2998	.605	.9911	.1018	27	.536	.6934	0.9301
156	.3688	.2672	01.104	.9938	.1867	24	31.035	.7404	.7539
159	.3791	.2344	.603	.9944	.2704	21	.534	.7808	.5712
162	.3881	.2013	02.102	.9930	.3527	18	32.033	.8140	.3836
165	.3956	.1680	.601	.9896	.4339	15	.532	.8384	+ .1908
168	.4018	.1347	03.100	.9842	.5134	12	33.031	.8536	— .0035
171	.4066	.1011	.598	.9770	.5915	9	.529	.8586	.1977
174	.4100	.0675	04.097	.9678	.6681	7	34.028	.8528	.3894
177	.4121	.0338	.596	.9566	.7430	3	.527	.8359	.5760
+ 180	— 1.4128	+ 0.0000	1905.095	— 1.9436	+ 1.8163	— 0	1935.027	+ 0.8078	— 0.7549

If extreme precision is required in future comparisons of photographic observations the changes to the elements A, B, F, and G, computed earlier in this paper, may be applied. For this reason the quantities

$$X = \cos E - e, \quad Y = (1-e)^{\frac{1}{2}} \sin E$$

have been given in Table V. For negative M reverse the sign of Y. It may be assumed that the elements are for 1915, which is near the weighted mean of the epochs of the normal places.

The desiderata for future advance of our knowledge of this interesting system are continued spectroscopic observation of both stars, photographic observation of the kind done at Potsdam and Königsberg, in that part of the orbit where the distance is not too small, abundant micrometer measures especially near periastron. As the system is on Professor SCHLESINGER's list of test objects for parallax observers, the parallax is likely to become better known than it is now; it would be desirable if parallax observers included a determination of the mass ratio.

APPENDIX.

For the convenience of a future worker on this system a list of measures not contained in NØRLUND's and ABETTI's lists is here appended; also some measures received after the completion of the normal places. There are some minor differences between NØRLUND's list and mine, chiefly in the names of observers and number of nights, not sufficiently important to give them here.

			°	″
1845.75	Smyth	1?	136.1	2.8
1846.01	Hind	5,1	133.9	2.79
1847.08	Hind	3	132.4	
1853.38	Peters	3	124.2	3.06
1854.12	Powell	9	117.3	
1855.10	Powell	12	116.6	
1863.33	Adolph	6	96.7	2.70
1868.19	Williams	1	80.4	1.85
1872.28	Brünnow	2	24.2	1.32
1873.23	Brünnow	8,6	3.2	1.12
1877.33	Pritchett	1	291.5	1.35
1877.47	Jedrzejewicz	4	294.2	1.48
1878.26	Seabroke etc.	2	285.8	1.62
1879.37	Seabroke etc.	3	280.6	1.67
1880.42	Seabroke etc.	1	271.8	1.80
1881.33	Perry	9	269.7	1.88
1883.38	Küstner	6,5	258.0	1.96
1885.48	Baillaud	1	245.3	2.22
1888.34	Celoria	15	224.5	1.88
1888.34	Robinson	1	221.8	1.62
1889.42	Celoria	6	216.8	1.64
1890.12	Giacomelli	5,2	207.9	1.65
1890.40	Celoria	4	210.3	1.56
1891.18	Byers	1	204.9	1.62
1891.23	Dennis	1	201.3	1.72
1891.36	Bellamy	1	199.7	1.79
1891.38	Wickham	1	204.1	1.82
1891.39	Robinson	2	201.4	1.60
1893.34	Lewis	4	187.7	1.74
1894.25	Hough	1	181.8	1.61
1894.34	Schiaparelli	6	182.2	1.75
1895.30	Hough	2	173.0	1.88

			°	″
1897.41	Celoria	2	164.5	2.09
1898.50	Celoria	2	158.4	2.05
1900.40	Celoria	3	153.1	2.32
1900.44	Fayet	2	151.4	2.20
1903.38	Celoria	3	140.9	2.38
1903.39	Wirtz	3	141.3	2.45
1904.43	Celoria	16	139.4	2.29
1905.21	Farman	1?	134.4	2.29
1905.43	Celoria	9	136.7	2.61
1906.43	Wirtz	1	133.9	2.83
1907.36	Comstock	3	129.8	2.81
1907.36	Guillaume	2	132.3	2.80
1907.37	Wirtz	2	130.4	2.72
1908.18	Roe	1	130.2	2.30
1908.32	Comstock	3	129.1	2.86
1909.08	Hertzsprung...	phot.	128.2	2.76
1909.24	Roe	3	125.5	2.28
1909.39	Comstock	3	125.4	2.80
1909.40	Phillips	2	124.9	2.97
1910.13	Callisen	2	124.0	3.04
1912.35	Comstock	2	120.0	2.94
1913.21	Chapman	1	120.2	3.22
1913.27	van Biesbroeck	5	119.2	2.92
1913.34	Bowyer	3	117.4	2.94
1913.35	Slater	1	118.4	3.19
1913.42	Comstock	2	117.6	2.99
1914.13	Chapman	1	119.8	3.50
1914.17	Doolittle	2	115.0	3.20
1914.21	Guillaume	1	117.1	3.04
1914.24	Rabe	6	114.8	3.00
1914.28	Doberck	5	116.8	3.08
1914.28	Bowyer	2	115.0	2.84
1914.28	van Biesbroeck	3	116.6	2.78
1914.38	Comstock	3	115.1	2.96
1914.38	Jones	1	116.2	3.01
1915.00	Stearns	phot.	113.3	3.12
1915.05	Brown	4	116.0	3.10
1915.21	Doberck	3	115.3	3.13
1915.27	Rabe	12	114.5	3.24
1915.29	Jones	3	115.1	2.87
1915.30	van Biesbroeck	3	116.5	3.02
1915.35	Stearns	phot.	114.2	3.17
1915.39	Comstock	3	114.6	3.03
1916.19	van Biesbroeck	4	113.0	2.89
1916.23	Rabe	8	111.5	3.10
1916.27	Pavel	2	112.4	3.06
1916.29	Bernewitz	1	111.5	2.75
1916.30	Comstock	3	110.9	3.01
1916.32	Doberck	3	111.5	2.96
1916.34	Stearns	phot.	111.0	2.99
1917.03	Stearns	phot.	110.0	3.16
1917.17	Doolittle	5	108.3	3.07
1917.22	Comstock	4	111.6	3.08
1917.26	Franks	3	110.6	3.06
1917.28	Doberck	3	111.3	3.15
1917.35	Pettit	3	111.3	3.23
1917.36	Stearns	phot.	109.3	3.06
1918.00	Stearns	phot.	107.7	2.98
1918.07	van Biesbroeck	3	107.9	2.82
1918.24	Doberck	4	108.2	2.95
1918.32	Aitken	1	104.7	2.87
1918.34	Comstock	3	106.9	2.92
1918.37	Vos v. Steenwyk	4	110.8	3.20
1919.23	Vos v.Steenwyk	1	106.0	3.09
1919.24	van Biesbroeck	3	106.4	2.88
1919.25	Comstock	3	107.2	3.07
1919.26	Aitken	2	104.2	2.81
1919.31	Doberck	5,4	107.0	3.01
1919.39	Lord	4	104.7	3.24
1919.42	Furner	2	105.9	2.99
1919.43	Leavenworth ..	2	106.8	2.77
1920.14	Doberck	4,3	104.8	2.96
1920.22	Storey	1	104.3	3.42
1920.23	van Biesbroeck	2	103.7	2.74
1920.32	Chandon	7	103.7	2.97
1920.34	Stearns	phot.	101.9	2.88
1920.40	Leavenworth ..	4	103.6	2.86
1921.17	Haarh	1	103.0	3.01
1921.17	Luplau Janssen	1	104.1	2.95
1921.18	Stearns	phot.	100.7	2.83
1921.18	van den Bos ..	3	102.5	2.79
1921.28	Bernewitz	2	101.3	2.75

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1921.30	Doberck	3	103.0	2.76
1921.33	Przybyllok	6	100.9	3.07
1921.37	Labitzke	2	101.2	3.10
1921.38	Leavenworth ..	5	101.4	2.70
1921.39	Jackson	1	100.1	3.45
1921.45	van Biesbroeck	3	101.3	2.54
1922.23	Doberck	4,3	99.2	2.85
1922.24	Witchell	3	98.0	2.76
1922.25	Labitzke	2	95.8	3.12
1922.29	van den Bos ..	3	98.4	2.56
1922.37	Leavenworth ..	3	98.7	2.74
1922.39	Cullen	1	100.8	2.57
1922.40	Przybyllok ...	21,20	99.4	2.91
1923.18	G. Struve	4	97.3	2.47
1923.23	Dick	4	97.2	2.80
1923.25	van den Bos ..	6	96.5	2.58
1923.34	Krumpholz ...	2	95.6	2.82
1923.36	van Biesbroeck	3	96.9	2.31
1923.40	Przybyllok ...	17	96.0	2.71
1923.41	Leavenworth ..	5	96.0	2.54
1923.48	Labitzke	5	94.5	2.78
1924.09	G. Struve	2	93.4	2.41
1924.13	Dick	6	93.6	2.62
1924.13	van Biesbroeck	4	94.4	2.33
1924.24	van den Bos ..	35	93.5	2.33
1924.35	Lauritzen	6	94.8	2.33
1924.36	Luplau Janssen	6	94.6	2.45
1924.37	Przybyllok ...	14	94.4	2.65
1924.43	Fjeltofte	1	97.9	2.74
1924.53	Leavenworth ..	3	93.7	2.46
1925.03	Guillaume	1	93.4	2.52
1925.16	Doberck	3	90.3	2.56
1925.19	van Biesbroeck	4	90.5	2.12
1925.30	van den Bos ..	40	89.6	2.17
1925.39	Luplau Janssen	17	88.9	2.07
1925.39	Fjeltofte	2	89.3	2.52
1925.40	Lauritzen	19	89.3	2.04
1925.41	Leavenworth ..	6	89.6	2.30
1925.93	Phillips	4	86.3	2.22
1926.19	van Biesbroeck	4	88.0	1.95
1926.31	Voûte	6	87.4	2.23

Not included in normal places:

1924.26	L. Mc. Cormick	phot.	95.7	2.34
1925.31	Richardson[1] ..	1	....	2.26
1925.62	Luplau Janssen	1	88.0	2.28
1925.62	Lauritzen	1	88.3	2.31
1926.16	Doberck	4	87.1	2.27
1926.37	Lauritzen	3	86.3	2.17
1926.38	Leavenworth ..	9	86.2	2.11
1926.42	Luplau Janssen	6	85.6	2.30
1927.37	Richardson[1] ..	3	....	1.88

[1] diffraction micrometer.